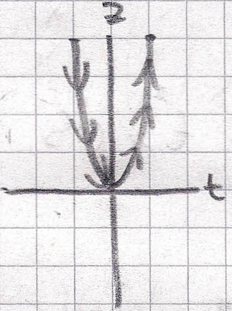
TALLER 10

① $z = 2x^2 + y^2$ y el plano $y = \sqrt{2}x$ debajo de $z = 4$

$z = 2x^2 + 2x^2$

$z = 4x^2$



$4 = 4t^2$

$t^2 = 1$

$t^2 \leq 1$

$-1 \leq t \leq 1$

Parametrización

Si $x = t$

$$\begin{cases} x = t \\ y = \sqrt{2}\, t \\ z = 4t^2 \end{cases} \qquad -1 \leq t \leq 1$$

$$L = \int_{-1}^{1} \sqrt{1 + (\sqrt{2})^2 + (8t)^2}$$

$$L = \int_{-1}^{1} \sqrt{64t^2 + 3}$$

$$L = 2\int_0^1 \sqrt{64\left(t^2 + \frac{3}{64}\right)}$$

Sustitución trigonométrica

$$\int_0^1 16 \sqrt{\left(\frac{\sqrt{3}}{8}\right)^2 + t^2}$$

$$a = \frac{\sqrt{3}}{8} \tan\theta$$

$$16\int_0^1 \sqrt{\left(\frac{\sqrt{3}}{8}\right)^2 + \left(\frac{\sqrt{3}}{8}\right)^2 \tan^2\theta}$$

Límites de integración

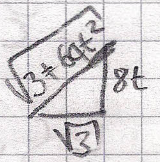
$1 = \frac{\sqrt{3}}{8} \tan\theta \qquad 0 = \tan\theta \frac{\sqrt{3}}{8}$

$\frac{8}{\sqrt{3}} = \tan\theta \qquad 0$

$\tan^{-1}(8/\sqrt{3}) = \theta_1$

$$\frac{\sqrt{3}}{8}16\int_0^1 \sqrt{1 + \tan^2\theta}$$

$$2\sqrt{3}\int_0^1 \sec\theta \; d\theta$$



$$2\sqrt{3} \;\ln|\sec\theta + \tan\theta| + c$$

$$2\sqrt{3} \;\ln\left|\frac{\sqrt{3+64t^2}}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right|$$

(2) $x^2+y^2=4$ $(2,0,0)$ 2π

$x=2\cos t$
$y=2\operatorname{sen} t$
$z=\omega t+c$

$\begin{cases} x=2\cos t \\ y=2\operatorname{sen} t \\ z=t \end{cases}$ $0\leq t\leq 2\pi$

$0=0+c$
$c=0$

$z(2\pi)=2\pi$
$\omega(2\pi)=2\pi$
$\omega=1$

$$m=\int_0^{2\pi} t \, ds$$

$ds=\|v'(t)\|$

$\begin{pmatrix} -2\operatorname{sen}(t) \\ 2\cos(t) \\ 1 \end{pmatrix}$

$\sqrt{4\operatorname{sen}^2(t)+4\cos^2(t)+1}$

$\sqrt{5}$

$$\int_0^{2\pi} t\sqrt{5}$$

$\frac{4\pi^2}{2}\cdot\sqrt{5}$ $\boxed{2\pi^2\cdot\sqrt{5}}$

(3) $x^2+(y-1)^2=1$

$x=a(t-\operatorname{sen} t)$

$y=a(1-\cos t)$

$x=t-\operatorname{sen} t$

$y=1-\cos t$

$$L=\int_C \sqrt{(1-\cos t)^2+(\operatorname{sen} t)^2}$$

$$L=\int_C \sqrt{1-2\cos t+\cos^2 t+\operatorname{sen}^2 t}$$

$$L = \int_C \sqrt{1 - 2\cos t + 1}$$

identidad

$$\cos^2\left(\frac{x}{2}\right) = \frac{1 + \cos x}{2}$$

$$L = \int_C \sqrt{2 - 2\cos t}\, dt$$

$$\int_C \sqrt{2(1 - \cos t)}\, dt$$

$$\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$$

$$\int_C \sqrt{4 \operatorname{sen}^2 (x/2)}$$

$$\int_C 2 \operatorname{sen} (x/2)$$

$$-4 \cos (x/2) \Big|_0^{2\pi}$$

$$4 \left[-\cos(\pi) + \cos(0) \right]$$

$$4 \left[-(-1) + 1 \right]$$

$$4 [2]$$

$$\boxed{8}$$

4. Sea C la recta que une a (1,1,1) con (2,3,1) y $f(x,y,z) = xy^2 + e^z$ Halle

$$\int_C f(x,y,z)$$

Parametrización

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + t \begin{pmatrix} x_f - x_0 \\ y_f - y_0 \\ z_f - z_0 \end{pmatrix}$$

$$x = 1 + t(1)$$
$$y = 1 + t(2)$$
$$z = 1 + t(0)$$

$$\left\{ \begin{array}{l} x = 1+t \\ y = 1+2t \quad 0 \leq t \leq 1 \\ z = 1 \end{array} \right.$$

$$r(t) = \begin{pmatrix} 1+t \\ 1+2t \\ 1 \end{pmatrix}$$

$$r'(t) = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \quad \|r'(t)\| = \sqrt{5}$$

$$\int_0^1 f(x,y,z)\, ds$$

$$ds = \|r'(t)\|$$

$$\int_0^1 xy^2 + e^z \quad \|r'(t)\| dt$$

$$\int_0^1 \left[(1+t)(1+2t)^2 + e^1\right] \sqrt{5}\, dt$$

$$\int_0^1 \left[(1+t)(1+4t+4t^2) + e^1\right] \sqrt{5}\, dt$$

$$\int_0^1 \left[1 + 4t + 4t^2 + t + 4t^2 + 4t^3 + e^1\right] \sqrt{5}\, dt$$

$$\int_0^1 \left[4t^3 + 8t^2 + 5t + 1 + e^1\right] \sqrt{5}\, dt$$

$$t^4\sqrt{5} + \frac{\sqrt{5}\,8t^3}{3} + \frac{\sqrt{5}t^2}{2} + \sqrt{5}t + \sqrt{5}e^1 t \Big|_0^1$$

$$\sqrt{5} + \frac{\sqrt{5}\,8}{3} + \frac{\sqrt{5}}{2} + \sqrt{5} + \sqrt{5}e^1$$

$$\sqrt{5}\left[\frac{8}{3} + \frac{1}{2} + 1 + e^1\right]$$

$$\boxed{\frac{25\sqrt{5}}{6} + \sqrt{5}e^1}$$

⑤ $\vec{F} = \dfrac{K}{x^2+y^2+z^2}$ → Esto es la magnitud

Multiplicamos por la dirección y la norma

→ Dirigida al origen

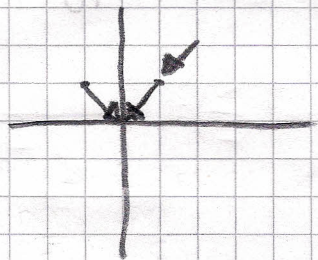
$$\vec{F} = \frac{K}{x^2+y^2+z^2} \cdot \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \frac{K}{\sqrt{x^2+y^2+z^2}}$$

$$\vec{F} = \begin{pmatrix} \dfrac{-Kx}{(x^2+y^2+z^2)^{3/2}} \\ \dfrac{-Ky}{(x^2+y^2+z^2)^{3/2}} \\ \dfrac{-Kz}{(x^2+y^2+z^2)^{3/2}} \end{pmatrix}$$

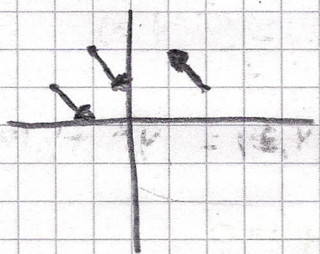
$\int \vec{F}$

⑥ $F(x,y) = \langle -x, -y \rangle$

$(-1,1) = (1,-1)$



$F(x,y) = \langle y, x \rangle$



$(2,1) = (1,2)$

$(-2,1) = (1,-2)$

$(-1,2) = (2,-1)$

⑦ Sean $\vec{F}(x,y,z) = \begin{pmatrix} 2xy^2+z \\ 2x^2y+2y \\ x \end{pmatrix} = \begin{matrix} dx \\ dy \\ dz \end{matrix}$

Si es conservativo (rot=0) podemos hallar una F que para F(r(b)) - F(r(a))

$\vec{r}(t) = \begin{pmatrix} \operatorname{Sen}^8\left(\frac{\pi t}{2}\right)+1 \\ t^3 \\ t^8 \end{pmatrix} \quad 0 \leq t \leq 1$

$F_x = 2xy^2+z$
$F_y = 2x^2y+2y$
$F_z = x$

$$\begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ 2xy^2+z & 2x^2y+2y & x \end{vmatrix}$$

$$\begin{pmatrix} 0-0 \\ -1+1 \\ 4xy-4xy \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Conservativo

Integramos f_x

$$\int 2xy^2+z \, dx$$

$$F = x^2y^2 + zx + g(y,z)$$

derivamos con respecto a y

$$2yx^2 + g'(y,z)$$

Igualamos f_y

$$2x^2y + 2y = 2yx^2 + g'(y,z)$$

$$\int 2y = \int g'(y,z)$$

$$g(y,z) = y^2 + g(z)$$

$$F = x^2y^2 + zx + y^2 + g(z)$$

~~dx~~ derivamos con respecto a z

$$F = \cancel{x} + g'(z)$$

Igualamos a f_z

$$x = x + g'(z)$$

$$g'(z) = 0$$

$$g(z) = K$$

$$F = x^2+y^2+zx+y^2+\underbrace{K}_{=0}$$

$$F(r(b)) - F(r(a))$$

$$(sen^8(\pi t/2)+1)^2 + t^6 + t^8(sen^8(\pi t/2)) + t^2$$

$$4+1+1+1 - 0$$

8 Calcule

$$\int_C \overset{M}{(3+2xy)}\,dx + \overset{N}{(x^2-3y^2)}\,dy$$

va $(0,1)$ a $(0,-e^{\pi})$

Si es conservativo hay una F que evaluamos en $F(r(b)) - F(r(a))$

$M_y = 2x$

$N_x = 2x$

$M_y = N_x$ Conservativo

$f_x = 3 + 2xy$ | $f_y = x^2 - 3y^2$

$\int 3 + 2xy\, dx$ | $\cancel{x^2} + g'(y) = \cancel{x^2} - 3y^2$

$\frac{d}{dy}\; 3x + x^2 y + g(y)$ | $\int g'(y) = \int -3y^2$

$x^2 + g'(y)$ | $g(y) = -y^3$

$F = 3x + x^2 y - y^3$

Parametrizacion

$y = 1 + (t)(-e^{\pi} - 1)$ | $y = -te^{\pi} - t + 1$

$x = 0 + (t)(0)$ | $x = 0$

$F(r(b))^{\to 1} - F(r(a))^{\to 0}$

$-y^3$

$-[\pi - te^{\pi} - t]^3 + [1] =$ $\boxed{e^{3\pi^2}+1}$

$-[1 - e^{\pi} - 1]^3 + 1 =$

⑨ Sea C la curva con parametrización

$$\bar{r}(t) = \begin{pmatrix} t \\ \text{sen } t \\ t^2 \cos t \end{pmatrix} \quad 0 \leq t \leq \pi$$

Sea $f(x,y,z) = z^2 e^{x^2 y} + x^2$ y $\vec{F} = \nabla f$

Halle $\int_C \vec{F}\, d\vec{r}$ ↳ conservativo

Si ∇f es $\vec{F}$ entonces f es la función original y usamos TFIL

$F(r(b)) - F(r(a))$

$t^4 \cos^2 t \cdot e^{t^2 \cdot \text{sen}(t)} + t^2$

$F(r(b))^{\pi} - F(r(a))^{\to 0} \to 0$

$\pi^4 \cdot 1 + \pi^2$ $\boxed{\pi^4 + \pi^2}$

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$$S=\int_a^b \|r'(t)\|$$

11 Escriba la longitud de arco de la curva intersección del cilindro

$x^2+(y-1)^2=1$ y el paraboloide

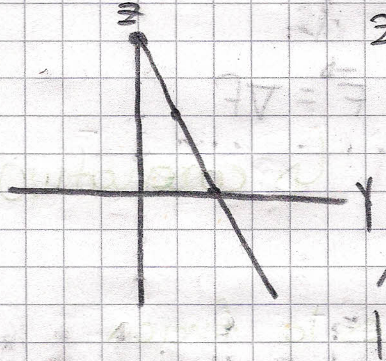
$$z=4-x^2-y^2$$

$$z=4-(x^2+y^2)$$

$x^2+y^2-2y+1=1$

$x^2+y^2=2y$

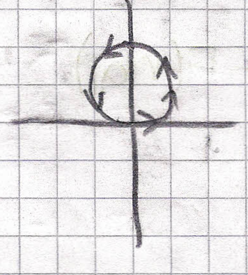
$$S=\int_a^b \|r'(t)\|$$



$$z=4-2y$$

$$r(t)=\begin{cases} x=\cos(\theta)\\ y=1+\operatorname{sen}(\theta)\\ z=4-2-2\operatorname{sen}\theta \end{cases}$$

$$r(t)=\begin{cases} x=\cos(\theta)\\ y=1+\operatorname{sen}(\theta)\\ z=2-2\operatorname{sen}(\theta) \end{cases}$$



$$\int_0^{2\pi}\sqrt{\operatorname{sen}^2(\theta)+\cos^2(\theta)+4\cos^2(\theta)}$$

$$\int_0^{2\pi}\sqrt{1+4\cos^2\theta}$$